

## Lista de exercícios 4

### Vetores, operações e bases ordenadas

**Exercício 1.** Para cada um dos vetores  $\vec{u}$  e  $\vec{v}$  abaixo, calcule  $\vec{u} + \vec{v}$ ,  $\vec{u} - \vec{v}$ ,  $2\vec{u} - 3\vec{v}$ ,  $\|\vec{u}\|$ ,  $\|\vec{v}\|$ ,  $\|\vec{u} + \vec{v}\|$ ,  $\|\vec{u} - \vec{v}\|$ ,  $\vec{u} \cdot \vec{v}$  e  $(\vec{u} + \vec{v}) \cdot (\vec{u} - \vec{v})$ . Normalize os vetores  $\vec{u}$  e  $\vec{v}$ .

$$(a) \begin{aligned} \vec{u} &= (3, -1) \\ \vec{v} &= (1, 4) \end{aligned}$$

$$(f) \begin{aligned} \vec{u} &= (\sqrt{3}, -\sqrt{5}) \\ \vec{v} &= (0, 1) \end{aligned}$$

$$(k) \begin{aligned} \vec{u} &= (0, -1, -\sqrt{5}) \\ \vec{v} &= (1, 2, 0) \end{aligned}$$

$$(b) \begin{aligned} \vec{u} &= (3, 0) \\ \vec{v} &= (1, 0) \end{aligned}$$

$$(g) \begin{aligned} \vec{u} &= (\frac{1}{2}, -1) \\ \vec{v} &= (1, \frac{2}{3}) \end{aligned}$$

$$(l) \begin{aligned} \vec{u} &= (1, 1, 1) \\ \vec{v} &= (2, 2, 2) \end{aligned}$$

$$(c) \begin{aligned} \vec{u} &= (1, -1) \\ \vec{v} &= (-1, 1) \end{aligned}$$

$$(h) \begin{aligned} \vec{u} &= (2, -3, 1) \\ \vec{v} &= (-1, 0, 4) \end{aligned}$$

$$(m) \begin{aligned} \vec{u} &= (0, 0, 1) \\ \vec{v} &= (0, -1, 0) \end{aligned}$$

$$(d) \begin{aligned} \vec{u} &= (1, 2) \\ \vec{v} &= (3, 4) \end{aligned}$$

$$(i) \begin{aligned} \vec{u} &= (1, -1, 1) \\ \vec{v} &= (1, 2, 2) \end{aligned}$$

$$(n) \begin{aligned} \vec{u} &= (1, -1, 0) \\ \vec{v} &= (-1, -1, 0) \end{aligned}$$

$$(e) \begin{aligned} \vec{u} &= (3, 1) \\ \vec{v} &= (2, -4) \end{aligned}$$

$$(j) \begin{aligned} \vec{u} &= (1, 0, 1) \\ \vec{v} &= (1, 2, 0) \end{aligned}$$

$$(o) \begin{aligned} \vec{u} &= (1, 0, -1) \\ \vec{v} &= (1, \sin(\pi/4), -\cos(5\pi/6)) \end{aligned}$$

**Exercício 2.** Dados os vetores  $\vec{u}$  e  $\vec{v}$  abaixo, determine se  $\{\vec{u}, \vec{v}\}$  é base de  $\mathbb{R}^2$ . Em caso afirmativo, determine as coordenadas do vetor  $\vec{w} = (1, -1)$  na base ordenada  $\mathfrak{B} = (\vec{u}, \vec{v})$ .

$$(a) \begin{aligned} \vec{u} &= (3, -1) \\ \vec{v} &= (1, 4) \end{aligned}$$

$$(c) \begin{aligned} \vec{u} &= (1, -1) \\ \vec{v} &= (-1, 1) \end{aligned}$$

$$(e) \begin{aligned} \vec{u} &= (\sqrt{3}, -\sqrt{5}) \\ \vec{v} &= (0, 1) \end{aligned}$$

$$(b) \begin{aligned} \vec{u} &= (3, 0) \\ \vec{v} &= (1, 0) \end{aligned}$$

$$(d) \begin{aligned} \vec{u} &= (1, 2) \\ \vec{v} &= (1, -1) \end{aligned}$$

$$(f) \begin{aligned} \vec{u} &= (\frac{1}{2}, -1) \\ \vec{v} &= (1, \frac{2}{3}) \end{aligned}$$

**Exercício 3.** Dados os vetores  $\vec{u}, \vec{v}, \vec{w}$  abaixo, determine se  $\{\vec{u}, \vec{v}, \vec{w}\}$  é base de  $\mathbb{R}^3$ . Em caso afirmativo, determine as coordenadas de  $\vec{w} = (1, 0, -1)$  na base ordenada  $\mathfrak{B} = (\vec{u}, \vec{v}, \vec{w})$ .

$$(a) \begin{aligned} \vec{u} &= (2, -3, 1) \\ \vec{v} &= (-1, 0, 4) \\ \vec{w} &= (1, 1, 1) \end{aligned}$$

$$(c) \begin{aligned} \vec{u} &= (1, 0, 1) \\ \vec{v} &= (2, 4, 0) \\ \vec{w} &= (1, -2, 2) \end{aligned}$$

$$(e) \begin{aligned} \vec{u} &= (1, 1, 1) \\ \vec{v} &= (\frac{2}{3}, \frac{2}{3}, \frac{2}{3}) \\ \vec{w} &= (1, -1, 1) \end{aligned}$$

$$(b) \begin{aligned} \vec{u} &= (1, -1, 1) \\ \vec{v} &= (1, 2, 2) \\ \vec{w} &= (1, \frac{1}{2}, \frac{3}{2}) \end{aligned}$$

$$(d) \begin{aligned} \vec{u} &= (0, -1, -\sqrt{5}) \\ \vec{v} &= (1, 2, 0) \\ \vec{w} &= (1, 0, -1) \end{aligned}$$

$$(f) \begin{aligned} \vec{u} &= (1, -1, 0) \\ \vec{v} &= (-1, -1, 0) \\ \vec{w} &= (1, 1, 0) \end{aligned}$$

**Exercício 4.** Em cada um dos casos abaixo, determine se os pontos  $A, B, C, D$  dados são vértices de um paralelogramo.

$$(a) A(1, 2), B(4, 6), \\ C(7, 8) \text{ e } D(4, 4)$$

$$(d) A(-1, 2), B(3, 5), \\ C(6, 3) \text{ e } D(2, 0)$$

$$(g) A(0, 0, 0), B(2, 1, 1), \\ C(4, 3, 2) \text{ e } D(2, 2, 1)$$

$$(b) A(1, 0), B(3, 1), \\ C(5, 4) \text{ e } D(2, 3)$$

$$(e) A(1, 1), B(4, 5), \\ C(7, 4) \text{ e } D(4, 0)$$

$$(h) A(-1, 2, 3), B(3, 5, 6), \\ C(5, 8, 9) \text{ e } D(1, 5, 6)$$

$$(c) A(0, 0), B(3, 1), \\ C(5, 4) \text{ e } D(2, 3)$$

$$(f) A(1, 2, 3), B(4, 5, 6), \\ C(7, 8, 9) \text{ e } D(4, 5, 3)$$

$$(i) A(1, 1, 1), B(3, 4, 5), \\ C(6, 7, 8) \text{ e } D(4, 4, 4)$$