Lista de exercícios 4
Vetores, operações e bases ordenadas

Exercício 1. Para cada um dos vetores $\vec{u}$ e $\vec{v}$ abaixo, calcule $\vec{u}+\vec{v}$, $\vec{u}-\vec{v}$, $2\vec{u}-3\vec{v}$, $\|\vec{u}\|$, $\|\vec{v}\|$, $\|\vec{u}+\vec{v}\|$, $\|\vec{u}-\vec{v}\|$, $\vec{u}\cdot\vec{v}$ e $(\vec{u}+\vec{v})\cdot(\vec{u}-\vec{v})$. Normalize os vetores $\vec{u}$ e $\vec{v}$.

(a) $\vec{u}=(3,-1)$, $\vec{v}=(1,4)$

(b) $\vec{u}=(3,0)$, $\vec{v}=(1,0)$

(c) $\vec{u}=(1,-1)$, $\vec{v}=(-1,1)$

(d) $\vec{u}=(1,2)$, $\vec{v}=(3,4)$

(e) $\vec{u}=(3,1)$, $\vec{v}=(2,-4)$

(f) $\vec{u}=(\sqrt{3},-\sqrt{5})$, $\vec{v}=(0,1)$

(g) $\vec{u}=(\frac{1}{2},-1)$, $\vec{v}=(1,\frac{2}{3})$

(h) $\vec{u}=(2,-3,1)$, $\vec{v}=(-1,0,4)$

(i) $\vec{u}=(1,-1,1)$, $\vec{v}=(1,2,2)$

(j) $\vec{u}=(1,0,1)$, $\vec{v}=(1,2,0)$

(k) $\vec{u}=(0,-1,-\sqrt{5})$, $\vec{v}=(1,2,0)$

(l) $\vec{u}=(1,1,1)$, $\vec{v}=(2,2,2)$

(m) $\vec{u}=(0,0,1)$, $\vec{v}=(0,-1,0)$

(n) $\vec{u}=(1,-1,0)$, $\vec{v}=(-1,-1,0)$

(o) $\vec{u}=(1,0,-1)$, $\vec{v}=\big(1,\,\mathrm{sen}(\pi/4),-\cos(5\pi/6)\big)$

Exercício 2. Dados os vetores $\vec{u}$ e $\vec{v}$ abaixo, determine se $\{\vec{u},\vec{v}\}$ é base de $\mathbb{R}^2$. Em caso afirmativo, determine as coordenadas do vetor $\vec{w}=(1,-1)$ na base ordenada $\mathfrak{B}=(\vec{u},\vec{v})$.

(a) $\vec{u}=(3,-1)$, $\vec{v}=(1,4)$

(b) $\vec{u}=(3,0)$, $\vec{v}=(1,0)$

(c) $\vec{u}=(1,-1)$, $\vec{v}=(-1,1)$

(d) $\vec{u}=(1,2)$, $\vec{v}=(1,-1)$

(e) $\vec{u}=(\sqrt{3},-\sqrt{5})$, $\vec{v}=(0,1)$

(f) $\vec{u}=(\frac{1}{2},-1)$, $\vec{v}=(1,\frac{2}{3})$

Exercício 3. Dados os vetores $\vec{u},\vec{v},\vec{w}$ abaixo, determine se $\{\vec{u},\vec{v},\vec{w}\}$ é base de $\mathbb{R}^3$. Em caso afirmativo, determine as coordenadas de $\vec{w}=(1,0,-1)$ na base ordenada $\mathfrak{B}=(\vec{u},\vec{v},\vec{w})$.

(a) $\vec{u}=(2,-3,1)$, $\vec{v}=(-1,0,4)$, $\vec{w}=(1,1,1)$

(b) $\vec{u}=(1,-1,1)$, $\vec{v}=(1,2,2)$, $\vec{w}=(1,\frac{1}{2},\frac{3}{2})$

(c) $\vec{u}=(1,0,1)$, $\vec{v}=(2,4,0)$, $\vec{w}=(1,-2,2)$

(d) $\vec{u}=(0,-1,-\sqrt{5})$, $\vec{v}=(1,2,0)$, $\vec{w}=(1,0,-1)$

(e) $\vec{u}=(1,1,1)$, $\vec{v}=(\frac{2}{3},\frac{2}{3},\frac{2}{3})$, $\vec{v}=(1,-1,1)$

(f) $\vec{u}=(1,-1,0)$, $\vec{v}=(-1,-1,0)$, $\vec{w}=(1,1,0)$

Exercício 4. Em cada um dos casos abaixo, determine se os pontos A,B,C,D dados são vértices de um paralelogramo.

(a) $A(1,2)$, $B(4,6)$, $C(7,8)$ e $D(4,4)$

(b) $A(1,0)$, $B(3,1)$, $C(5,4)$ e $D(2,3)$

(c) $A(0,0)$, $B(3,1)$, $C(5,4)$ e $D(2,3)$

(d) $A(-1,2)$, $B(3,5)$, $C(6,3)$ e $D(2,0)$

(e) $A(1,1)$, $B(4,5)$, $C(7,4)$ e $D(4,0)$

(f) $A(1,2,3)$, $B(4,5,6)$, $C(7,8,9)$ e $D(4,5,3)$

(g) $A(0,0,0)$, $B(2,1,1)$, $C(4,3,2)$ e $D(2,2,1)$

(h) $A(-1,2,3)$, $B(3,5,6)$, $C(5,8,9)$ e $D(1,5,6)$

(i) $A(1,1,1)$, $B(3,4,5)$, $C(6,7,8)$ e $D(4,4,4)$